(3) Standard Deviation

M Ltd $= \frac{(220-220)+20}{220} \times 100 = 9.09\%$

$= \frac{(250-220)+20}{220} \times 100 = 22.73\%$

$= \frac{(280-220)+20}{220} \times 100 = 36.36\%$

N Ltd $= \frac{(290-290)+3.5}{290} \times 100 = 1.25\%$

$= \frac{(310-290)+3.5}{220} \times 100 = 8.10\%$

$= \frac{(330-290)+3.5}{290} \times 100 = 15\%$

P	X	P (X)	$(X-\bar{X})$	$(X-\bar{X})^2$ P	Y	P (Y)	$(Y-\bar{Y})$	$(Y-\bar{Y})^{2/3}$
0.2	9.09	1.82	-15.01	45.06	1.21	0.24	- 7.582	11.50
0.5	22.73	11.37	-1.37	0.94	8.10	4.04	- 0.692	0.24
0.3	36.36	10.91	12.26	45.09	15	4.5	6.208	11.56
	$\sum$P (X) =	24.10		91.09		8.792		23.30

σx $= \sqrt{91.09} = 9.54\%$

σy $= \sqrt{23.30} = 4.83\%$

Standard Deviation of M Ltd. is more than Standard Deviation of N Ltd.

Hence, stock of M Ltd. is more risky.

EFFICIENT & OPTIMUM PORTFOLIO

Question – 09

Following is the data regarding six securities:

	A	B	C	D	E	F
Return (%)	8	8	12	4	9	8
Risk (Standard deviation)	4	5	12	4	5	6

(i) Assuming three will have to be selected, state which ones will be picked.

(ii) Assuming perfect correlation, show whether it is preferable to invest 75% in A and 25% in C or to invest 100% in E

(SM TYK – 05)

Solution:

(i) Efficient Securities

- Security B & F are inefficient because Security A provides same return at lower level of risk.
- Security D is inefficient because security A provides higher return at the same level of risk

(ii) 75% in A & 25% in C

ER_p = (8 × 0.75) + (12 × 0.25) = 9%

SD_p = (4 × 0.75) + (12 + 0.25) = 6%

100 % in E

ER = 9%

S.D = 5%

Investment in 100% in stock E is better than portfolio because Stock E Provides same return at Lower Level Risk.

Question – 10

Suppose that economy A is growing rapidly and you are managing a global equity fund and so far you have invested only in developed-country stocks only. Now you have decided to add stocks of economy A to your portfolio. The table below shows the expected rates of return, standard deviations, and correlation coefficients (all estimates are for aggregate stock market of developed countries and stock market of Economy A).

	Developed Country Stocks	Stocks of Economy A
Expected rate of return (annualized percentage)	10	15
Risk [Annualized Standard Deviation (%)]	16	30
Correlation Coefficient (ρ)	0.30	

Assuming the risk-free interest rate to be 3%, you are required to determine:

(a) What percentage of your portfolio should you allocate to stocks of Economy A if you want to increase the expected rate of return on your portfolio by 0.5%?

(b) What will be the standard deviation of your portfolio assuming that stocks of Economy A are included in the portfolio as calculated above?

(c) Also show how well the Fund will be compensated for the risk undertaken due to inclusion of stocks of Economy A in the portfolio?

(SM TYK – 24)

Solution:

(1) Calculation of weights

$ERP = ER_A \times W_A + ER_B W_B$

$10.5 = 10W_A + 15(1 - W_A)$

$10.5 = 10W_A + 15 - 15W_A$

$W_A = \frac{4.5}{5} = 0.9$

$W_B = 0.1$

Developed Country = 90%

Economy A = 10%

(2) SD of Portfolio

$$\sigma_p = \sqrt{\sigma_A^2 W_A^2 + \sigma_B^2 W_B^2 + 2 \times \sigma_A \times W_A \times \sigma_B \times W_B \times r_{AB}}$$

$$= \sqrt{16^2 \times 0.9^2 + 30^2 \times 0.10^2 + 2 \times 16 \times 0.9 \times 30 \times 0.10 \times 0.3}$$

$= 15.56\%$

(3) Sharpe ratio

$$SR = \frac{ER - RF}{\sigma}$$

Develop country only = $\frac{10-3}{16}$ = 0.4375

Portfolio = $\frac{10.5-3}{15.56}$ = 0.482

Sharpe ratio will improve.

MINIMUM VARIANCE PORTFOLIO

Question – 11

An investor has two portfolios known to be on minimum variance set for a population of three securities A, B and C having below mentioned weights:

	WA	WB	WC
Portfolio X	0.30	0.40	0.30
Portfolio Y	0.20	0.50	0.30

It is supposed that there are no restrictions on short sales.

(i) What would be the weight for each stock for a portfolio constructed by investing ₹ 5,000 in portfolio X and ₹ 3,000 in portfolio Y?.

(ii) Suppose the investor invests ₹ 4,000 out of ₹ 8,000 in security A. How he will allocate the balance between security B and C to ensure that his portfolio is on minimum variance set?

(SM TYK – 31)

Solution:

(I) **PORTFOLIO (8,000)**

Portfolio X (5,000)

A 5,000 × 0.3 = 1,500

B 5,000 × 0.4 = 2,000

C 5,000 × 0.3 = 1,500

Portfolio Y (3,000)

A 3,000 × 0.20 = 600

B 3,000 × 0.50 = 1,500

C 3,000 × 0.30 = 900

$$W_A = \frac{1,500 + 600}{8,000} = 0.2625$$

$$W_B = \frac{2{,}000 + 1{,}500}{8{,}000} = 0.4375$$

$$W_C = \frac{1{,}500 + 900}{8{,}000} = 0.30$$

(II) Calculation of W_B & W_C if W_A = 0.5

Y = a + bx

0.4 = a + 0.30b ....(i)

0.5 = a + 0.20b ....(ii)

-0.1 = 0.1b

$$b = \frac{-0.1}{0.1} = -1 \text{ Slope}$$

Put Slope in equation (i)

0.4 = a + 0.30 × -1

0.4 = a – 0.30

a = 0.70

Equation

Y = 0.70 + -1x

If W_A = 0.5

W_B = 0.70 (-1 × 0.5)

= 0.2

W_C = 1 – 0.5 – 0.2 = 0.3

Allocation = A = 8,000 × 0.5 = 4,000

B = 8,000 × 0.2 = 1,600

C = 8,000 × 0.3 = 2,400

Question – 12

An Investor is proposing to invest ₹ 10,000/- in two Portfolios A and B in the ratio of 3:2. The Portfolios have three securities each with following weights:

	Wx	Wy	Wz
Portfolio A	0.30	0.25	0.45
Portfolio B	0.20	0.45	0.35

You are required to

(i) Calculate the weight of each stock.

(ii) Calculate the amount allocated to B and C if half of the funds are allocated to security X.

Solution:

1. Calculation of Weights

10,000 in 3:2	X	Y	Z
(6,000) Portfolio A	1,800	1,500	2,700
(4,000) Portfolio B	800	1,800	1,400
	2,600	3,300	4,100

Wx = 2600/10000 = 0.26

Wy = 3,300/10,000 = 0.33

Wz = 4,100/10,000 = 0.41

2. Calculation of Wy & Wz when Wx = 0.5 :

Using Critical Line:

$Y = a + bx$

$0.25 = a + 0.30$ ……….(i)

$0.45 = a + 0.20$ ……….(ii)

$-0.20 = 0.1b$

$$b = \frac{-0.20}{0.1} = -2$$

Putting b value of m in equation (i)

$0.25 = a + (-2 \times 0.30)$

$a = 0.85$

Equation:

$y = 0.85 - 2x$

$Wx = 0.85 (-2 \times 0.5)$

$Wx = -0.15$

$Wz = 1 - (0.5 - 0.15)$

$= 0.65$

Investment:

X = 10,000 × 0.5 = 5,000

Y = 10,000 × (-0.15) = (1,500) Short sale.

Z = 10,000 × 0.65 = 6,500

PART II: CAPITAL ASSET PRICING MODEL

CALCULATION OF BETA

Question – 13

The distribution of return of security 'F' and the market portfolio 'P' is given below:

Probability	Return %	
	F	P
0.30	30	-10
0.40	20	20
0.30	0	30

You are required to calculate the expected return of security 'F' and the market portfolio 'P', the covariance between the market portfolio and security and beta for the security.

(SM TYK – 09)

Solution:

Calculation of Cov_{xy}

p	x	p(x)	$(x - \bar{x})$	$(x - \bar{x})^2$ p	y	P(y)	$(y - \bar{y})$	$(y - \bar{y})^2$ p	$(x - \bar{x})(y - \bar{y})$ p
0.30	30	9	13	50.7	-10	-3	-24	172.80	-93.60
0.40	20	8	3	3.60	20	8	6	14.40	7.20
0.30	0	0	-17	86.70	30	9	16	76.80	-81.60
	$\bar{x}$ =	17%		141(%)²	$\bar{y}$ =	14%		264(%)²	168(%)²

$$x = \sqrt{141(\%)^2} = 11.87\%$$

$$\sigma_y = \sqrt{264(\%)^2} = 16.25\%$$

$$\beta = \frac{Cov_{xm}}{\sigma_m^2} = \frac{168(\%)^2}{264(\%)^2} = 0.636$$

Question – 14

Given below is information of market rates of Returns and Data from two Companies A and B:

	Year 2007	Year 2008	Year 2009
Market (%)	12.0	11.0	9.0
Company A (%)	13.0	11.5	9.8
Company B (%)	11.0	10.5	9.5

You are required to determine the beta coefficients of the shares of Company A and Company B.

(SM TYK – 10)

Solution:

Beta of Portfolio in case of A & Market

Year	x	$(x - \bar{x})$	$(x - \bar{x})^2$	y	$(y - \bar{y})$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
1	13	1.57	2.46	12	1.33	1.77	2.09
2	11.5	0.07	0.0049	11	0.33	0.11	0.02

3	9.8	-1.63	2.66	9	-1.67	2.79	2.72
	34.3		**5.12**	**32**		**4.67**	**4.83**

$\bar{x}$ = 11.43%

$\sigma_x = \sqrt{\frac{5.12}{3}} = 1.31\%$

$\bar{y}$ = 10.67%

$\sigma_y = \sqrt{\frac{4.67}{3}} = 1.25\%$

$Cov_{xy} = \frac{4.83}{3} = 1.61(\%)^2$

$B_P = \frac{1.31}{1.25} \times 0.983 = 1.03$

$r_{xm} = \frac{1.61}{1.31 \times 1.25} = 0.983$

Beta of Portfolio in case of B & Market

Year	x	$(x-\bar{x})$	$(x-\bar{x})^2$	y	$(y-\bar{y})$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
1 2 3	11 10.5 9.5	0.67 0.17 -0.83	0.45 0.03 0.69	12 11 9	1.33 0.33 -1.67	1.77 0.11 2.79	0.89 0.06 1.39
	31		**1.17**	**32**		**4.67**	**2.34**

$\bar{\bar{x}}$ = 10,33%

$\sigma_x = \sqrt{\frac{1.17}{3}} = 0.62\%$

$\bar{y}$ = 10.67%

$\sigma_y = \sqrt{\frac{4.67}{3}} = 1.25\%$

$$Cov_{xy} = \frac{2.34}{3} = 0.78(\%)^2$$

$$r_{xy} = \frac{0.78}{0.62 \times 1.25} = 1.006$$

$$B_p = \frac{0.62}{0.125} \times 1.006 = 0.50$$

Question – 15

The rates of return on the security of Company X and market portfolio for 10 periods are given below:

Period	Return of Security X (%)	Return on market portfolio (%)
1	20	22
2	22	20
3	25	18
4	21	16
5	18	20
6	-5	8
7	17	-6
8	19	5
9	-7	6
10	20	11

(i) What is the beta of Security X?

(ii) What is the characteristic line for Security X?

(SM TYK – 12)

Solution:

Year	x	$(x - \bar{x})$	$(x - \bar{x})^2$	y	$(y - \bar{y})$	$(Y - \bar{Y})^2$	$(x - \bar{x})(y - \bar{y})$
1	20	5	25	22	10	100	50
2	22	7	49	20	8	64	56
3	25	10	100	18	6	36	60
4	21	6	36	16	4	16	24
5	18	3	9	20	8	64	24
6	-5	-20	400	8	-4	16	80
7	17	2	4	-6	-18	324	-36
8	19	4	16	5	-7	49	-28